Sequences and Series- Questions

June 2019 Mathematics Advanced Paper 1: Pure Mathematics 1

1.

A competitor is running a 20 kilometre race.

She runs each of the first 4 kilometres at a steady pace of 6 minutes per kilometre.
After the first 4 kilometres, she begins to slow down.

In order to estimate her finishing time, the time that she will take to complete each subsequen kilometre is modelled to be 5% greater than the time that she took to complete the previous kilometre.

Using the model,

(a) show that her time to run the first 6 kilometres is estimated to be 36 minutes 55 seconds,
(2)

(b) show that her estimated time, in minutes, to run the rth kilometre, for $5 \leqslant r \leqslant 20$, is

$$6 \times 1.05^{r-4}$$

(1)

(c) estimate the total time, in minutes and seconds, that she will take to complete the race.
(4)

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2.

A company, which is making 140 bicycles each week, plans to increase its production. The number of bicycles produced is to be increased by d each week, starting from 140 in week 1, to $140 + d$ in week 2, to $140 + 2d$ in week 3 and so on, until the company is producing 206 in week 12.

(*a*) Find the value of d.

(2)

After week 12 the company plans to continue making 206 bicycles each week.

(*b*) Find the total number of bicycles that would be made in the first 52 weeks starting from and including week 1.

(5)

3.

$a_1 = 1$ A sequence $a_1, a_2, a_3, \ldots$ is defined by

$$a_{n+1} = \frac{k(a_n + 1)}{a_n}, \qquad n \geqslant 1$$

where k is a positive constant.

(a) Write down expressions for a_2 and a_3 in terms of k, giving your answers in their simplest form.

(3)

Given that $\sum_{r=1}^{3} a_r = 10$

(b) find an exact value for k.

(3)

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4.

A sequence $a_1, a_2, a_3, \ldots$ is defined by

$$a_1 = 4,$$

$$a_{n+1} = 5 - ka_n, \quad n \geq 1,$$

where k is a constant.

(a) Write down expressions for a_2 and a_3 in terms of k.

(2)

Find

(b) $\sum_{r=1}^{3} (1 + a_r)$ in terms of k, giving your answer in its simplest form,

(3)

(c) $\sum_{r=1}^{100} (a_{r+1} + ka_r)$.

(1)

5.

On John's 10th birthday he received the first of an annual birthday gift of money from his uncle. This first gift was £60 and on each subsequent birthday the gift was £15 more than the year before. The amounts of these gifts form an arithmetic sequence.

(*a*) Show that, immediately after his 12th birthday, the total of these gifts was £225.

(1)

(*b*) Find the amount that John received from his uncle as a birthday gift on his 18th birthday.

(2)

(*c*) Find the total of these birthday gifts that John had received from his uncle up to and including his 21st birthday.

(3)

When John had received n of these birthday gifts, the total money that he had received from these gifts was £3375.

(*d*) Show that $n^2 + 7n = 25 \times 18$.

(3)

(*e*) Find the value of n, when he had received £3375 in total, and so determine John's age at this time.

(2)

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6.

Jess started work 20 years ago. In year 1 her annual salary was £17 000. Her annual salary increased by £1500 each year, so that her annual salary in year 2 was £18 500, in year 3 it was £20 000 and so on, forming an arithmetic sequence. This continued until she reached her maximum annual salary of £32000 in year k. Her annual salary then remained at £32 000.

(*a*) Find the value of the constant k.

(2)

(*b*) Calculate the total amount that Jess has earned in the 20 years.

(5)

7.

(i) A sequence $U_1, U_2, U_3, \ldots$ is defined by

$$U_{n+2} = 2U_{n+1} - U_n, \quad n \geq 1,$$

$$U_1 = 4 \text{ and } U_2 = 4.$$

Find the value of

(a) U_3,

(1)

(b) $\sum_{n=1}^{20} U_n$.

(2)

(ii) Another sequence $V_1, V_2, V_3, \ldots$ is defined by

$$V_{n+2} = 2V_{n+1} - V_n, \quad n \geq 1,$$

$V_1 = k$ and $V_2 = 2k$, where k is a constant.

(a) Find V_3 and V_4 in terms of k.

(2)

Given that $\sum_{n=1}^{5} V_n = 165$,

(b) find the value of k.

(3)

8.

A sequence of numbers $a_1, a_2, a_3 \ldots$ is defined by

$$a_{n+1} = 5a_n - 3, \qquad n \geq 1.$$

Given that $a_2 = 7$,

(*a*) find the value of a_1.

(2)

(*b*) Find the value of $\sum_{r=1}^{4} a_r$.

(3)

9.

In the year 2000 a shop sold 150 computers. Each year the shop sold 10 more computers than the year before, so that the shop sold 160 computers in 2001, 170 computers in 2002, and so on forming an arithmetic sequence.

(*a*) Show that the shop sold 220 computers in 2007.

(2)

(*b*) Calculate the total number of computers the shop sold from 2000 to 2013 inclusive.

(3)

In the year 2000, the selling price of each computer was £900. The selling price fell by £20 each year, so that in 2001 the selling price was £880, in 2002 the selling price was £860, and so on forming an arithmetic sequence.

(*c*) In a particular year, the selling price of each computer in £s was equal to three times the number of computers the shop sold in that year. By forming and solving an equation, find the year in which this occurred.

(4)

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10.

A sequence $a_1, a_2, a_3, \ldots$ is defined by

$$a_1 = 4,$$
$$a_{n+1} = k(a_n + 2), \qquad \text{for } n \geq 1$$

where k is a constant.

(a) Find an expression for a_2 in terms of k.

(1)

Given that $\sum_{i=1}^{3} a_i = 2$,

(b) find the two possible values of k.

(6)

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11.

4. A sequence $u_1, u_2, u_3, \ldots$, satisfies

$$u_{n+1} = 2u_n - 1, \quad n \geq 1.$$

Given that $u_2 = 9$,

(a) find the value of u_3 and the value of u_4 ,

(2)

(b) evaluate $\sum_{r=1}^{4} u_r$.

(3)

12.

7. Lewis played a game of space invaders. He scored points for each spaceship that he captured.

Lewis scored 140 points for capturing his first spaceship.

He scored 160 points for capturing his second spaceship, 180 points for capturing his third spaceship, and so on.

The number of points scored for capturing each successive spaceship formed an arithmetic sequence.

(a) Find the number of points that Lewis scored for capturing his 20th spaceship.

(2)

(b) Find the total number of points Lewis scored for capturing his first 20 spaceships.

(3)

Sian played an adventure game. She scored points for each dragon that she captured. The number of points that Sian scored for capturing each successive dragon formed an arithmetic sequence.

Sian captured n dragons and the total number of points that she scored for capturing all n dragons was 8500.

Given that Sian scored 300 points for capturing her first dragon and then 700 points for capturing her nth dragon,

(c) find the value of n.

(3)

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13.

5. A sequence of numbers $a_1, a_2, a_3, \ldots$ is defined by

$$a_1 = 3,$$

$$a_{n+1} = 2a_n - c, \quad (n \geq 1),$$

where c is a constant.

(a) Write down an expression, in terms of c, for a_2.

(1)

(b) Show that $a_3 = 12 - 3c$.

(2)

Given that $\sum_{i=1}^{4} a_i \geq 23$,

(c) find the range of values of c.

(4)

14.

6. A boy saves some money over a period of 60 weeks. He saves 10p in week 1, 15p in week 2, 20p in week 3 and so on until week 60. His weekly savings form an arithmetic sequence.

(*a*) Find how much he saves in week 15.

(2)

(*b*) Calculate the total amount he saves over the 60 week period.

(3)

The boy's sister also saves some money each week over a period of m weeks. She saves 10p in week 1, 20p in week 2, 30p in week 3 and so on so that her weekly savings form an arithmetic sequence. She saves a total of £63 in the m weeks.

(*c*) Show that

$$m(m + 1) = 35 \times 36.$$

(4)

(*d*) Hence write down the value of m.

(1)

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15.

4. A sequence $x_1, x_2, x_3, \ldots$ is defined by

$$x_1 = 1,$$

$$x_{n+1} = a x_n + 5, \qquad n \geq 1,$$

where a is a constant.

(*a*) Write down an expression for x_2 in terms of a.

(1)

(*b*) Show that $x_3 = a^2 + 5a + 5$.

(2)

Given that $x_3 = 41$

(*c*) find the possible values of a.

(3)

16.

9. A company offers two salary schemes for a 10-year period, Year 1 to Year 10 inclusive.

Scheme 1: Salary in Year 1 is £P.
Salary increases by £($2T$) each year, forming an arithmetic sequence.

Scheme 2: Salary in Year 1 is £($P + 1800$).
Salary increases by £T each year, forming an arithmetic sequence.

(*a*) Show that the total earned under Salary Scheme 1 for the 10-year period is

$$£(10P + 90T).$$

(2)

For the 10-year period, the total earned is the same for both salary schemes.

(*b*) Find the value of T.

(4)

For this value of T, the salary in Year 10 under Salary Scheme 2 is £29 850.

(*c*) Find the value of P.

(3)

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17.

5. A sequence a_1, a_2, a_3, ..., is defined by

$$a_1 = k,$$

$$a_{n+1} = 5a_n + 3, \quad n \geq 1,$$

where k is a positive integer.

(*a*) Write down an expression for a_2 in terms of k.

(1)

(*b*) Show that $a_3 = 25k + 18$.

(2)

(*c*) (i) Find $\sum_{r=1}^{4} a_r$ in terms of k, in its simplest form.

(ii) Show that $\sum_{r=1}^{4} a_r$ is divisible by 6.

(4)

18.

9. (*a*) Calculate the sum of all the even numbers from 2 to 100 inclusive,

$$2 + 4 + 6 + \dots\dots + 100.$$

(3)

(*b*) In the arithmetic series

$$k + 2k + 3k + \dots\dots + 100,$$

k is a positive integer and k is a factor of 100.

(i) Find, in terms of k, an expression for the number of terms in this series.

(ii) Show that the sum of this series is

$$50 + \frac{5000}{k}.$$

(4)

(*c*) Find, in terms of k, the 50th term of the arithmetic sequence

$$(2k + 1),\quad (4k + 4),\quad (6k + 7),\ \dots\ ,$$

giving your answer in its simplest form.

(2)

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19.

4. A sequence a_1, a_2, a_3, ... is defined by

$$a_1 = 2,$$

$$a_{n+1} = 3a_n - c$$

where c is a constant.

(*a*) Find an expression for a_2 in terms of c.

(1)

Given that $\sum_{i=1}^{3} a_i = 0$,

(*b*) find the value of c.

(4)

20.

6. An arithmetic sequence has first term a and common difference d. The sum of the first 10 terms of the sequence is 162.

(a) Show that $10a + 45d = 162$.

(2)

Given also that the sixth term of the sequence is 17,

(b) write down a second equation in a and d,

(1)

(c) find the value of a and the value of d.

(4)

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21.

5. A sequence of positive numbers is defined by

$$a_{n+1} = \sqrt{(a_n^2 + 3)}, \quad n \geq 1,$$

$$a_1 = 2.$$

(a) Find a_2 and a_3, leaving your answers in surd form.

(2)

(b) Show that $a_5 = 4$.

(2)

22.

9. A farmer has a pay scheme to keep fruit pickers working throughout the 30 day season. He pays £a for their first day, £$(a + d)$ for their second day, £$(a + 2d)$ for their third day, and so on, thus increasing the daily payment by £d for each extra day they work.

A picker who works for all 30 days will earn £40.75 on the final day.

(a) Use this information to form an equation in a and d.

(2)

A picker who works for all 30 days will earn a total of £1005.

(b) Show that $15(a + 40.75) = 1005$.

(2)

(c) Hence find the value of a and the value of d.

(4)

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23.

7. Jill gave money to a charity over a 20-year period, from Year 1 to Year 20 inclusive. She gave £150 in Year 1, £160 in Year 2, £170 in Year 3, and son on, so that the amounts of money she gave each year formed an arithmetic sequence.

(*a*) Find the amount of money she gave in Year 10.
(2)

(*b*) Calculate the total amount of money she gave over the 20-year period.
(3)

Kevin also gave money to charity over the same 20-year period.

He gave £A in Year 1 and the amounts of money he gave each year increased, forming an arithmetic sequence with common difference £30.

The total amount of money that Kevin gave over the 20-year period was **twice** the total amount of money that Jill gave.

(*c*) Calculate the value of A.
(4)

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24.

9. The first three terms of a geometric sequence are

$$7k - 5, \qquad 5k - 7, \qquad 2k + 10$$

where k is a constant.

(*a*) Show that $11k^2 - 130k + 99 = 0$
(4)

Given that k is not an integer,

(*b*) show that $k = \frac{9}{11}$
(2)

For this value of k,

(*c*) (i) evaluate the fourth term of the sequence, giving your answer as an exact fraction,

(ii) evaluate the sum of the first ten terms of the sequence.
(6)

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25.

1. A geometric series has first term a and common ratio $r = \frac{3}{4}$.

The sum of the first 4 terms of this series is 175.

(*a*) Show that $a = 64$.

(2)

(*b*) Find the sum to infinity of the series.

(2)

(*c*) Find the difference between the 9th and 10th terms of the series.
Give your answer to 3 decimal places.

(3)

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26.

5. (i) All the terms of a geometric series are positive. The sum of the first two terms is 34 and the sum to infinity is 162.

Find

(*a*) the common ratio,

(4)

(*b*) the first term.

(2)

(ii) A different geometric series has a first term of 42 and a common ratio of $\frac{6}{7}$.

Find the smallest value of n for which the sum of the first n terms of the series exceeds 290.

(4)

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27.

6. The first term of a geometric series is 20 and the common ratio is $\frac{7}{8}$. The sum to infinity of the series is S_∞.

(a) Find the value of S_∞.

(2)

The sum to N terms of the series is S_N.

(b) Find, to 1 decimal place, the value of S_{12}.

(2)

(c) Find the smallest value of N, for which $S_\infty - S_N < 0.5$.

(4)

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28.

1. The first three terms of a geometric series are

$$18, 12 \text{ and } p$$

respectively, where p is a constant.

Find

(a) the value of the common ratio of the series,

(1)

(b) the value of p,

(1)

(c) the sum of the first 15 terms of the series, giving your answer to 3 decimal places.

(2)

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29.

3. A company predicts a yearly profit of £120 000 in the year 2013. The company predicts that the yearly profit will rise each year by 5%. The predicted yearly profit forms a geometric sequence with common ratio 1.05.

(*a*) Show that the predicted profit in the year 2016 is £138 915.

(1)

(*b*) Find the first year in which the yearly predicted profit exceeds £200 000.

(5)

(*c*) Find the total predicted profit for the years 2013 to 2023 inclusive, giving your answer to the nearest pound.

(3)

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30.

9. A geometric series is $a + ar + ar^2 + ...$

(*a*) Prove that the sum of the first n terms of this series is given by

$$S_n = \frac{a(1-r^n)}{1-r}$$

(4)

The third and fifth terms of a geometric series are 5.4 and 1.944 respectively and all the terms in the series are positive.

For this series find,

(*b*) the common ratio,

(2)

(*c*) the first term,

(2)

(*d*) the sum to infinity.

(3)

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31.

1. A geometric series has first term $a = 360$ and common ratio $r = \frac{7}{8}$.

Giving your answers to 3 significant figures where appropriate, find

(*a*) the 20th term of the series, **(2)**

(*b*) the sum of the first 20 terms of the series, **(2)**

(*c*) the sum to infinity of the series. **(2)**

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32.

6. The second and third terms of a geometric series are 192 and 144 respectively.

For this series, find

(*a*) the common ratio, **(2)**

(*b*) the first term, **(2)**

(*c*) the sum to infinity, **(2)**

(*d*) the smallest value of n for which the sum of the first n terms of the series exceeds 1000. **(4)**

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33.

3. The second and fifth terms of a geometric series are 750 and –6 respectively.

Find

(*a*) the common ratio of the series, **(3)**

(*b*) the first term of the series, **(2)**

(*c*) the sum to infinity of the series. **(2)**

34.

9. The adult population of a town is 25 000 at the end of Year 1.

A model predicts that the adult population of the town will increase by 3% each year, forming a geometric sequence.

(*a*) Show that the predicted adult population at the end of Year 2 is 25 750.

(1)

(*b*) Write down the common ratio of the geometric sequence.

(1)

The model predicts that Year N will be the first year in which the adult population of the town exceeds 40 000.

(*c*) Show that

$$(N-1)\log 1.03 > \log 1.6$$

(3)

(*d*) Find the value of N.

(2)

At the end of each year, each member of the adult population of the town will give £1 to a charity fund.

Assuming the population model,

(*e*) find the total amount that will be given to the charity fund for the 10 years from the end of Year 1 to the end of Year 10, giving your answer to the nearest £1000.

(3)